

	P.R.Government College (Autonomous) KAKINADA	Program&Semester III B.Sc. Mathematics Major & Statistics, Chemistry Minors (V Sem)			
CourseCode MAT-501 T	TITLEOFTHECOURSE Linear Algebra & Problem Solving Sessions				
Teaching	HoursAllocated:60(Theory)	L	T	P	C
Pre-requisites:	Advanced Calculus, Linear Algebra and Differential Equations	3	-	-	3

Course Objectives:

This course will cover the classical fundamental topics in numerical methods such as, approximation, numerical integration, numerical linear algebra, solution of nonlinear algebraic systems and solution of ordinary differential equations.

Course Outcomes:

On Completion of the course, the students will be able o-	
C01	understand the concepts of vector spaces, subspaces
C02	understand the concepts of basis, dimension and their properties
C03	understand the concept of linear transformation and its properties
C04	apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods
C0 5	learn the properties of inner product spaces and determine orthogonality in inner product spaces.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	
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Unit – 1: Vector Spaces-I

Vector Spaces - General properties of vector spaces - n-dimensional Vectors - addition and scalar multiplication of Vectors - internal and external composition - Null space - Vector subspaces -Algebra of subspaces - Linear Sum of two subspaces - linear combination of Vectors- Linear span Linear independence and Linear dependence of Vectors.

Unit – 2: Vector Spaces-II

Basis of Vector space - Finite dimensional Vector spaces - basis extension - co-ordinates- Dimension of a Vector space - Dimension of a subspace - Quotient space and Dimension of Quotient space.

Unit – 3: Linear Transformations

Linear transformations - linear operators- Properties of L.T- sum and product of L.Ts - Algebra of Linear Operators - Range and null space of linear transformation - Rank and Nullity of linear transformations - Rank- Nullity Theorem.

Unit – 4: Matrices

Characteristic equation - Characteristic Values - Characteristic vectors of a square matrix – Cayley Hamilton Theorem – problems on Cayley Hamilton Theorem.

Unit – 5: Inner product space

Inner product spaces- Euclidean and unitary spaces- Norm or length of a Vector- Schwartz inequality- Triangle Inequality- Parallelogram law- Orthogonality- Orthonormal set- Problems on Gram–Schmidt orthogonalisation process - Bessel's inequality.

Additional Inputs:

Echelon form and Normal form of a matrices, Consistent and inconsistent in Matrices.

III. References:

Text Books

1.Linear Algebra by J.N. Sharma and A.R. Vasishtha, published by Krishna Prakashan Media (P) Ltd.

2.Matrices by A.R.Vasishtha and A.K.Vasishtha published by Krishna Prakashan Media (P) Ltd.

Reference Books

1. Linear Algebra by Stephen H. Friedberg et. al. published by Prentice Hall of India Pvt.Ltd. 4 th Edition, 2007

2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson education low priced edition), New Delhi.

3. Matrices by Shanti Narayana, published by S.Chand Publications

IV Suggested Co-Curricular Activities:

Seminar/ Quiz/ Assignments/Applications of Linear Algebra in real life problems\ Problem Solving.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-V : PAPER-MAJOR XII & MINOR V

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Vector Spaces-I	2	1	20
II	Vector Spaces-II	2	1	20
III	Linear Transformations	1	1	15
IV	Matrices	1	2	25
V	Inner product space	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 3X10 = 30

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Total Marks = 50

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Pithapur Rajah's Government College (Autonomous), Kakinada

III Year B.Sc., Degree Examinations - V Semester

Mathematics Course Major XII & Minor V : LINEAR ALGEBRA

(Model Paper w.e.f. 2025-26)

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Time: 2Hrs

Max. Marks: 50

SECTION-A

Answer Any Three Questions, Selecting At Least One Question from Each Part.

Part – A

3 X 10 = 30

1. Essay question from Unit - I.
2. Essay question from Unit – I
3. Essay question from Unit - III.

Part – B

4. Essay question from Unit - IV .
1. Essay question from Unit - IV.
2. Essay question from Unit - V .

SECTION-B

Answer any four questions

4 X 5 M = 20 M

3. Short answer question from Unit - I.
4. Short answer question from Unit - I .
5. Short answer question from Unit – II.
6. Short answer question from Unit – II.
7. Short answer question from Unit - III.
8. Short answer question from unit – IV.
9. Short answer question from Unit – V.

P.R. GOVERNMENT COLLEGE (A), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank
PAPER–Major XII & Minor V: LINEAR ALGEBRA
Short answers

UNIT-I

1. Prove that the intersection of any two subspaces W_1 and W_2 of vector space $V(F)$ is subspace of $V(F)$.
2. Let p, q, r be the fixed elements of a field F . Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a vector sub space of $V_3(F)$.
3. Prove that the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.
4. Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the vectors $e_1 = (1, 1, 1)$, $e_2 = (1, 2, 3)$ and $e_3 = (2, -1, 1)$.
5. Show that the system of vectors $(1, 3, 2), (1, -7, -8), (2, 1, -1)$ of $V_3(R)$ is linearly dependent.
6. If α, β, γ are linearly independent vectors of $V(R)$, then show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.

UNIT-II

7. Show that the set of vectors $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ form a basis of $V_3(F)$.
8. Show that the set $\{(1,0,0), (1,1,0), (1,1,1)\}$ is a basis of $C^3(C)$. Hence find the coordinates of the vector $(3+4i, 6i, 3+7i)$ in $C^3(C)$.
9. Find the coordinates of α with respect to the basis set $\{x, y, z\}$ where $\alpha = (4, 5, 6)$, $x = (1, 1, 1)$, $y = (-1, 1, 1)$, $z = (1, 0, -1)$.
10. Prove that any two bases of a finite dimensional vector space $V(F)$ have the same number of elements.
11. If $U = \{(1,2,1), (0,1,2)\}$, $W = \{(1,0,0), (0,1,0)\}$ determine the dimension of $U + W$.

UNIT-III

12. The mapping $T: V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (x - y, x - z)$. Show that T is a linear transformation.
13. Show that the transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x - y, 0, y + z)$ is a linear transformation.
14. Find a linear transformation $T: R^3 \rightarrow R$ such that $T(1,1,1) = 3, T(0,1,-2) = 1, T(0,0,1) = -2$.

15. Find the null space, range, and nullity of the transformation $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (x + y, x - y, y)$.

UNIT-IV

16. Prove that the square matrices A and A' have the same characteristic values.
22. If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the characteristic values of a n -rowed square matrix A then $\lambda_1^2, \lambda_2^2, \dots, \lambda_n^2$ are the characteristic roots of A^2 .
23. Find the eigen values and eigen vectors of the square matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.
24. Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ by using Cayley-Hamilton theorem.

UNIT-V

25. State and Prove Triangle-Inequality.
26. State and prove Parallelogram law in an inner product space $V(F)$.
27. State and prove Parseval's inequality in an inner product space $V(F)$.
28. Prove that the set $S = \left\{ \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} \right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

Essay questions

UNIT-I

1. Let $V(F)$ be a vector space and $W \subseteq V$. Then necessary and sufficient condition for W to be a subspace of V is $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$
2. Prove that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.
3. If S and T are the subsets of a vector space $V(F)$ then prove that
(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ and (ii) $L(S \cup T) = L(S) + L(T)$.
4. Let $V(F)$ be a vector space and $S = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a finite subset of non-zero vectors of $V(F)$. Then prove that S is linearly dependent if and only if some vector $\alpha_k \in S, 2 \leq k \leq n$ can be expressed as a linear combination of its preceding vectors.

UNIT-II

5. State and Prove Basis Existence theorem.
6. Let W_1 and W_2 be two subspaces of a finite dimensional vector space $V(F)$. Then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

7. Let W be a subspace of a finite dimensional vector space $V(F)$ then prove that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.
8. Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a,b,c,d) / b - 2c + d = 0\}$, $W_2 = \{(a,b,c,d) / a = d, b = 2c\}$. Find the basis and dimension of i) W_1 ii) W_2 iii) $W_1 \cap W_2$ and hence find $\dim(W_1 + W_2)$.

UNIT-III

9. Let $U(F)$ and $V(F)$ be two vector spaces and $S = \{\alpha_1, \alpha_2, \alpha_3 \dots \alpha_n\}$ be a basis of U . Let $\{\delta_1, \delta_2, \dots \delta_n\}$ be a set of n vectors in V . then there exists a unique linear transformation $T : U \rightarrow V$ such that $T(\alpha_i) = \delta_i$ for $i = 1, 2, \dots n$.
10. Let $U(F)$ and $V(F)$ be two vector spaces and $T : U \rightarrow V$ is a linear transformation. Then prove that the range space $R(T)$ is a subspace of $V(F)$ and null space $N(T)$ is a subspace of $U(F)$.
11. State and prove Rank - Nullity theorem.
12. Let $T : R^3 \rightarrow R^3$ be defined by $T(x, y, z) = (x - y, 2y + z, x + y + z)$. Then verify Rank-nullity theorem.

UNIT-IV

13. Find the characteristic roots and the corresponding vectors of the square matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$
14. Find the characteristic roots and the corresponding vectors of the square matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$
15. State and prove Cayley-Hamilton theorem.
16. $A = \begin{pmatrix} 1 & 2 & -4 \\ 3 & -1 & 2 \\ 2 & 5 & 0 \end{pmatrix}$ verify Cayley-Hamilton theorem and hence find A^{-1} .
17. State Cayley -Hamilton theorem and use it to find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$.

UNIT-V

18. State and prove Cauchy- Schwarz's inequality.
19. State and prove Bessel's inequality.
20. Given $\{(2,1,3), (1,2,3), (1,1,1)\}$ is a basis of $R^3(R)$. Construct an orthonormal basis using Gram-Schmidt orthogonalization process.
21. Given $\{(1, -1, 2), (0, 2, 1), (1, 2, 0)\}$ is a basis of $R^3(R)$. Construct an orthonormal basis using Gram-Schmidt orthogonalization process.

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CourseCode MAT-501 P	TITLEOFTHECOURSE Linear Algebra & Problem Solving Sessions				
Teaching	HoursAllocated:30(Practicals)	L	T	P	C
Pre-requisites:		-	-	2	1

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Semester – V End Practical Examinations

Scheme of Valuation for Practical's

Time : 2 Hours

Max.Marks : 50

- **Record** - 10 Marks
- **Viva voce** - 10 Marks
- **Test** - 30 Marks
- **Answer any 5 questions. At least 2 questions from each section. Each question carries 6 marks.**

BLUE PRINT FOR PRACTICAL PAPER PATTERN

COURSE-Major XII & Minor V: Linear Algebra & Problem Solving Sessions

Unit	TOPIC	E.Q	Marks allotted to the Unit
I	Vector Space - I	2	12
II	Vector Space - II	1	06
III	Linear Transformation	2	12
IV	Matrices	2	12
V	Inner product spaces	1	06
	Total	08	48

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III year B.Sc., Degree Examinations - V Semester
Mathematics Course-Major XII & Minor V : Linear Algebra & Problem Solving Sessions
(w.e.f. 2023-24 Admitted Batch)
Practical Model Paper (w.e.f. 2025-2026)

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Time: 2Hrs

Max. Marks: 50M

Answer any 5 questions. At least 2 questions from each section.

5 x 6 = 30 Marks

SECTION - A

1. Unit - I.
2. Unit – I.
3. Unit - II.
1. Unit – III.

SECTION – B

2. Unit - III.
3. Unit - IV.
4. Unit – IV.
5. Unit - V.

- **Record** - 10 Marks
- **Viva voce** - 10 Marks